Density of States in Superconductor -Normal Metal-Superconductor Junctions

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Abstract

We consider the χ_0-dependence of the density of states inside the normal metal of a superconductor - normal metal - superconductor (SNS) junction. Here χ_0 is the phase difference of two superconductors of the junction. It is shown that in the absence of electron-electron interaction the energy dependence of the density of states has a gap which decreases as χ_0 increases and closes at $\chi_0 = \pi$. Both the analytical expressions for the χ_0 dependence of the density of states and the results of numerical simulations are presented.

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I. INTRODUCTION

The proximity effect in a normal metal near superconductor-normal metal boundary has been studied for many years. Recently, progress in microfabrication technology has revived the interest in this phenomenon. Due to Andreev reflections at superconductor-normal metal boundaries, the density of states inside the normal metal of a superconductor-normal metal (SN) junction is different from that of the bulk normal metal. This phenomenon has been studied both theoretically and experimentally[1−5]. In the superconductor - normal metal -

superconductor (SNS) junction it has been shown that at $\chi_0 = 0$ the Andreev reflections at the superconductor-normal metal interfaces cause the density of states to have a gap, which is of the order of the Thouless energy $E_c = D/L^2 \ll \Delta_0$. Here χ_0 is the phase difference between the two superconductors, Δ_0 is the modulus of the order parameter in the superconductors, L is the length of the normal metal of the junction (see Fig.1) and D is the diffusion constant in the metal.

In this paper we study the χ_0 dependence of the density of states $\nu(\epsilon, \chi_0)$ in the case when the length of the normal metal region is much larger than the electron elastic mean free path (diffusive regime). We study both analytically and numerically the behaviour of the density of states near the gap edge. This quantity can be measured for example in a tunneling experiment. We will show below that it also determines the conductance of the junction G_{SNS} at low temperatures T.

The density of states in the normal metal can be expressed in term of the retarded Green's function $g^R(\epsilon, x) = \cos\theta(\epsilon, x)$[6]

$$\nu(\epsilon) = \frac{\nu_0}{L}\int_{-L/2}^{L/2} \cos\theta_1(\epsilon, x)\cosh\theta_2(\epsilon, x)dx. \tag{1}$$

Here $\nu_0 = mp_F$ is the density of states in the bulk normal metal and p_F is the Fermi wave length; $\theta = \theta_1 + i\theta_2$ and $\chi = \chi_1 + i\chi_2$ are the complex variables described by Usadel equations[6],

$$\frac{D}{2}\partial_x{}^2\theta(\epsilon, x) + i\epsilon\sin\theta(\epsilon, x) - \frac{D}{4}(\partial_x\chi(\epsilon, x))^2\sin 2\theta(\epsilon, x) = \Delta_N(x)\cos\theta,$$
$$\partial_x\{\partial_x\chi(\epsilon, x)\sin^2\theta(\epsilon, x)\} = 0, \tag{2}$$

$$\Delta_N(x) = \gamma_N \int d\epsilon \cos\theta_1(\epsilon, x)\sinh\theta_2(\epsilon, x)\tanh(\epsilon/2kT). \tag{3}$$

Δ_N and $\gamma_N > 0$ are the modulus of the order parameter and the dimensionless repulsive interaction constant inside the normal metal.

The boundary conditions for Eq.2 are (we consider the case when the transmission coefficient of the normal metal-superconductor boundary $t = 1$),

$$\theta(\epsilon, x = \pm\frac{L}{2}) = \frac{\pi}{2},$$
$$\chi(\epsilon, x = \pm\frac{L}{2}) = \pm\frac{\chi_0}{2}. \tag{4}$$

Consider the situation when there is no electron-electron interaction in the normal metal and $\Delta_N = 0$. We will see that in this case the energy gap $E_g(\chi_0)$ of the electron spectrum is a decreasing function of χ_0 and is equal to zero only at $\chi_0 = \pi$ (See Fig. 2). In two limiting cases, when χ_0 is close to 0 or π we have

$$E_g(\chi_0) = E_c \begin{cases} C_2(1 - C_1\chi_0^2) & \chi_0 \ll \pi \\ C_3(\pi - \chi_0) & \pi - \chi_0 \ll \pi \end{cases} \tag{5}$$

II. THE ENERGY GAP

To get this result we take into account that at $\epsilon < E_g(\chi_0)$, $\theta_1 = \pi/2$ following Eq.1. In this case the second integral of Eq.2 gives the solution for $\theta_2(\epsilon, x)$:

$$\int_{\theta_2(\epsilon,x)}^{\theta_{20}} d\theta_2 \{\frac{\epsilon}{E_c}(\sinh\theta_{20} - \sinh\theta_2) + \alpha^2(\epsilon)(\cosh^{-2}\theta_2 - \cosh^{-2}\theta_{20})\}^{-\frac{1}{2}} = \frac{2x}{L}, \tag{6}$$

where $\theta_{20}(\epsilon)$ and $\alpha(\epsilon)$ are the functions of energy ϵ and χ_0 determined by the equations

$$\int_0^{\theta_{20}} d\theta_2 \{\frac{\epsilon}{E_c}(\sinh\theta_{20} - \sinh\theta_2) + \frac{\alpha^2(\epsilon)}{2}(\cosh^{-2}\theta_2 - \cosh^{-2}\theta_{20})\}^{-\frac{1}{2}} = 1,$$
$$\alpha(\epsilon)\int_0^{\theta_{20}} d\theta_2 \cosh^{-2}\theta_2 \{\frac{\epsilon}{E_c}(\sinh\theta_{20} - \sinh\theta_2) + \frac{\alpha^2(\epsilon)}{2}(\cosh^{-2}\theta_2 - \cosh^{-2}\theta_{20})\}^{-\frac{1}{2}} = \frac{\chi_0}{2}. \tag{7}$$

One can show that Eq.7 has solutions only at low energy ϵ. The energy gap $E_g(\chi_0)$ corresponds to the maximum value of ϵ, at which a solution exists. Beyond this value, $\theta_1 \neq \pi/2$ and therefore the density of states $\nu(\epsilon)$ is non zero.

Let us first consider the limiting case corresponding to $\chi_0 \ll \pi$. We can expand Eq.7 with respect to χ_0 and as a result we have the following equation:

$$\sqrt{\frac{\epsilon}{E_c}} = \int_0^{\theta_{20}} d\theta_2 (\sinh\theta_{20} - \sinh\theta_2)^{-\frac{1}{2}} - (\frac{\chi_0}{2})^2 A(\theta_{20}), \tag{8}$$

where A is of the order of unity. The right hand side of Eq.8 as a function of θ_{20} has a maxima equal to $\sqrt{C_2}(1 - C_1\chi_0^2/2)$. Here $C_{1,2}$ are numerical factors of order of unity. Their values can be obtained from the numerical solution of the Eq.2[7]: $C_1 = 0.91$ and $C_2 = 3.122$. At low energies when $\epsilon \ll E_c$, Eq.8 has a solution $\theta_{20} \sim \epsilon/E_c$ while at large energies $\epsilon > C_2 E_c(1 - C_1\chi_0^2)$, Eq.8 does not have a solution. Therefore in this limit $E_g = C_2 E_c(1 - C_1\chi_0^2)$, as given in Eq.5.

When $\chi_0 - \pi \ll \pi$, the energy gap is small, $E_g \ll E_c$ and we can expand Eq.7 with respect to the small parameter ϵ/E_c. As a result we have

$$\frac{\epsilon}{(\pi - \chi_0)E_c} = \pi^2 \tanh\theta_{20}\{8B_1 + (\pi - \chi_0)\pi B_2 \sinh\theta_{20}\}^{-1}. \tag{9}$$

where $B_{1,2}$ are also constants of the order of unity. Again the right hand side of Eq.9 as a function of θ_{20} reaches a maxima when

$$\cosh\theta_{20} = B_0(\pi - \chi_0)^{-\frac{1}{3}} \tag{10}$$

Therefore, at $\epsilon > C_3 E_c(\pi - \chi_0)$, Eq.9 has no solution and $E_g(\chi_0)$ is given by Eq.5. From the numerical solution we find $B_0 = 2.21$ and $C_3 = 2.43$. The insert of Fig.2 shows the linear dependence of the energy gap near $\chi_0 = \pi$.

III. THE GAP EDGE

Let us turn to the calculations of the ϵ-dependence of the density of states $\nu(\epsilon, \chi_0)$ at $\epsilon > E_g$.

In the region $\epsilon - E_g \ll E_g$, the quantities

$$\delta\theta_1(\epsilon, x) = \theta_1(\epsilon > E_g, x) - \frac{\pi}{2},$$
$$\delta\theta_2(\epsilon, x) = \theta_2(\epsilon > E_g, x) - \theta_2(E_g, x),$$

$$\delta\chi_1(\epsilon,x)=\chi_1(\epsilon>E_g,x)-\chi_1(E_g,x),$$
$$\delta\chi_2(\epsilon,x)=\chi_2(\epsilon>E_g,x), \tag{11}$$

are small and go to zero as ϵ approaches E_g. Here $\theta_2(E_g,x)$, $\chi_1(E_g,x)$ are the solution of Eq.2 at $\epsilon=E_g$ given by Eqs.6,7. When $\pi-\chi_0\ll\pi$

$$\sinh\theta_2(E_g,x)=\sinh\theta_{20}\cos\frac{\pi x}{L},$$
$$\partial_x\chi_1(E_g,x)=\frac{\pi\cosh\theta_{20}}{2L\cosh^2\theta_2(E_g,x)}, \tag{12}$$

where θ_{20} is given in Eq.10.

Expanding Eq.2 with respect to $\delta\theta_{1,2}$, $\delta\chi_{1,2}$ we get the following set of equations

$$\begin{aligned}\{\frac{D}{2}\partial_x^2+E_g\sinh\theta_2+\frac{D}{2}(\partial_x\chi_1)^2\cosh 2\theta_2\}\delta\theta_1-\frac{D}{2}\partial\chi_1\sinh 2\theta_2\partial_x\delta\chi_2\\ -\delta^3\theta_1(\frac{E_g}{6}\sinh\theta_2+\frac{1}{3}D(\partial_x\chi_1)^2\cosh 2\theta_2)\\ +\delta\theta_1(E_g\cosh\theta_2\delta\theta_2+D(\partial_x\chi_1)^2\sinh 2\theta_2\delta\theta_2+D\partial_x\delta\chi_1\partial_x\chi_1\cosh 2\theta_2-\frac{D}{2}(\partial_x\delta\chi_2)^2\cosh 2\theta_2)\\ +\delta^2\theta_1 D\partial_x\delta\chi_2\partial_x\chi_1\sinh 2\theta_2-D\partial_x\delta\chi_2\delta\theta_2\partial_x\chi_1\cosh 2\theta_2-\frac{D}{2}\partial_x\delta\chi_2\partial\delta\chi_1\sinh 2\theta_2\\ =-(\epsilon-\epsilon_g)\sinh\theta_2\delta\theta_1\end{aligned} \tag{13}$$

$$\begin{aligned}\{\frac{D}{2}\partial_x^2+E_g\sinh\theta_2+\frac{D}{2}(\partial_x\chi_1)^2\cosh 2\theta_2\}\delta\theta_2+\frac{D}{2}\partial_x\chi_1\sinh 2\theta_2\partial_x\delta\chi_1\\ -\delta^2\theta_1(\frac{E_g}{2}\cosh\theta_2+\frac{D}{2}(\partial_x\chi_1)^2\sinh 2\theta_2)+\delta\theta_1 D\partial_x\delta\chi_2\partial_x\chi_1\cosh 2\theta_2\\ -\frac{D}{4}(\partial_x\delta\chi_2)^2\sinh 2\theta_2=-(\epsilon-\epsilon_g)\cosh\theta_2\end{aligned} \tag{14}$$

$$\partial\{\partial_x\delta\chi_2\cosh^2\theta_2-\delta\theta_1\partial_x\chi_1\sinh 2\theta_2\}=0 \tag{15}$$

$$\partial_x\{\partial_x\delta\chi_1\cosh^2\theta_2+\delta\theta_2\partial_x\chi_1\sinh 2\theta_2-\delta\theta_1^2\partial_x\chi_1\cosh 2\theta_2+\delta\theta_1\partial_x\delta\chi_2\sinh 2\theta_2\}=0 \tag{16}$$

with the boundary conditions for $\delta\theta_{1,2},\delta\chi_{1,2}$:

$$\delta\theta_1(\epsilon,x=\pm\frac{L}{2})=\delta\theta_2(\epsilon,x=\pm\frac{L}{2})=0,$$
$$\delta\chi_1(\epsilon,x=\pm\frac{L}{2})=\delta\chi_2(\epsilon,x=\pm\frac{L}{2})=0. \tag{17}$$

Consider the linear part of Eq.13-17 at $\epsilon - E_g = 0$. Its solution determines the spatial dependence of $\delta\theta_{1,2}$, $\delta\chi_{1,2}$. Averaging Eq.13-17 over the sample and comparing the nonlinear terms in Eq.13 with right hand side term proportional to $(\epsilon - E_g)\sinh\theta_2\delta\theta_1$, we can determine the value of $\delta\theta_1$ as a function of $\epsilon - E_g$. As a result we have

$$\delta\theta_{10} \sim \sqrt{\frac{\epsilon - E_g(\chi_0)}{E_c}\cosh\theta_{20}C_4^2(\chi_0)} \tag{18}$$

where C_4 is a function of E_g and consequently of χ_0. Substituting Eqs.10, 18 into Eq.1 we obtain the asymptotic form valid when $\epsilon - E_g \ll E_g$:

$$\nu(\epsilon, \chi_0) = \nu_0 C_4(\chi_0)\sqrt{\frac{\epsilon - E_g(\chi_0)}{E_g(\chi_0)}} \tag{19}$$

with $C_4 \sim (\pi - \chi_0)^{-\beta}$, where $\beta \sim 2/3$. The value of β is a result of a numerical calculation over the whole energy range[7]. The square root energy dependence of the density of states is illustrated in Fig.3 which shows the square of the density of states as function of the energy very close to the energy gap. At higher energy (Fig.3 insert), we find that the density of states exhibits a smooth bump above the gap when phase difference is small. This smooth maximum turns to a sharper and sharper peak as the phase difference approaches π, i.e. as the gap closes.

It should be noted at this point that the asymptotic density of states at high energy or at $\chi_0 = \pi$ is smaller than the normal state value. The region near the superconductor provides a small contribution to the (spatially averaged) density of states. This apparent deficit of states is balanced by the excess density of states above the superconducting gap Δ_0 which according to our assumption is far above the energy range of interest. This deficit does not exist at the center of the normal metal or in a geometry where the relative area of the boundary between N and S goes to zero. The latter case is met in the billard geometry of Ref[4].

IV. DISCUSSION

Although the above results were derived for the junction geometry shown in Fig.1, the χ_0 and ϵ dependences of the density of states in Eqs.5,19 as well as the statement that the gap closes at $\chi_0 = \pi$ are general[9]. We believe that they are independent of the transmission coefficient of the superconductor-normal metal boundaries and the geometry of the normal region. The values of E_c and C_i, however, depend on these parameters. Similar conclusions were also reached in[4].

So far we assumed the electrons inside the normal metal do not interact with each other. In the presence of electron-electron interactions, Δ_N is not zero and one can study the interaction effects on the density of state by taking into account Δ_N in Eq.7 for E_g. Since $\gamma_N \ll 1$, one can carry out the perturbative calculation with respect to γ_N. As a result, at $\chi_0 \ll \pi$, the gap turns out to be smaller than that given in Eq.5 for the noninteracting case, i.e. $E_g(0) - E_g(\gamma_N) \sim \gamma_N E_c$. Furthermore, the gap closes at χ^* smaller than π

$$\pi - \chi^* \sim \gamma_N \tag{20}$$

Quantum fluctuations of the phase of the order parameter can also change the results derived above.

At finite temperature, in principle one also has to take into account the electron level broadening due to inelastic scattering. Such an inelastic process will introduce a temperature dependent density of state at the Fermi surface, i.e. $\nu(\epsilon = 0) \sim 1/E_c\tau_\epsilon$. However at $T \ll E_g(\chi_0)$, the inelastic scattering rate τ_ϵ^{-1} due to electron-phonon interaction is exponentially small.

Finally, let us calculate the conductance of the junction at $T \ll E_c$. The conductance G_{SNS} is the proportionality coefficient between the applied voltage and the disspative current averaged over the period of Josephson oscillations. At low temperature, the main contribution to G_{SNS} comes from the Debye relaxation mechanism[8]. The qualitative picture is the following. When voltage V is applied across the junction, the time dependence of χ_0 is determined by the Josephson relation

$$\frac{d\chi_0}{dt} = 2eV \tag{21}$$

At small V the time dependence of ν is determined by the corresponding time dependence of $\chi_0(t)$. In other words the quasiparticle energy levels move adiabatically with frequency $2eV$. The electron population of the energy levels follows the motion of the levels and as a result the electron distribution becomes nonequilibrium. The relaxation of the nonequilibrium distribution due to inelastic processes leads to the entropy production and therefore contributes to the conductance. As a result, we have[8],

$$G_{SNS} = \frac{e^2 v}{\hbar^2 \nu_0} \int d\epsilon \partial_\epsilon \tanh \frac{\epsilon}{2kT} \int_0^{2\pi} d\chi_0 \tau_\epsilon(\chi_0) \{ \int_{-\infty}^{\epsilon} d\epsilon' \frac{d\nu(\epsilon', \chi_0)}{d\chi_0} \}^2 \tag{22}$$

where $\tau_\epsilon(\chi_0)$ is the energy relaxation time of quasi particle of energy ϵ. In this case[10],

$$\frac{1}{\tau_\epsilon(\chi_0)} \sim \frac{\epsilon^3}{\Omega_D^2} \exp(-E_g(\chi_0)/kT) \tag{23}$$

is exponentially small in the time interval when $E_g(\chi_0(t)) \gg T$. Ω_D is the Debye frequency. On the other hand, the concentration of quasiparticles in this case is also exponentially small. These two exponential factors cancel each other and the main contribution to Eq.22 comes from the time interval when $E_g(\chi_0(t)) \sim T$. Since χ_0 changes linearly with time and $E_g(\chi_0)$ vanishes linearly as a function of $\chi_0 - \pi$ at χ_0 close to π, following Eq.22 we have

$$G_{SNS} \approx G_N C_4^2 (\frac{T}{E_c}) \tau_{in} T \tag{24}$$

Here $G_N = e^2 D \nu_0 \frac{S}{L}$ and $\tau_{in}^{-1} \sim T^3/\Omega_D^2$. C_4 is given in Eq.19. At $E_c \sim T$, Eq.24 matches the result of the conductance, $G_N \tau_{in} E_c^2/T$, which was obtained in[8] at the high temperature limit $T \gg E_c$.

Following the above arguments, the time dependence of the conductance has the form of narrow peaks with the amplitude of order of $G_N C_4^2(T/E_c)\tau_{in} E_c$ and duration of the order of $(eV)^{-1} T E_c$. This phenomenon can be connected with the well known $\cos \chi_0$ problem which has been investigated both experimentally and theoretically[11,12].

V. CONCLUSION

In conclusion we have shown that the density of states in the normal part of a SNS junction has a gap which closes when the superconducting phase difference is $\chi_0 = \pi$. The energy dependence of the density of states near the gap edge has been calculated. We should mention that nonequilibrium effects in superconductors which are connected with time dependence of quasiparticle spectrum have been considered in[13,14]. In these papers the enhancement of the critical current due to the nonequilibrium effects was considered. The effect considered there is proportional to $(eV\tau_\epsilon)^2$ while the DC current calculated above is linear in $(eV\tau_\epsilon)$.

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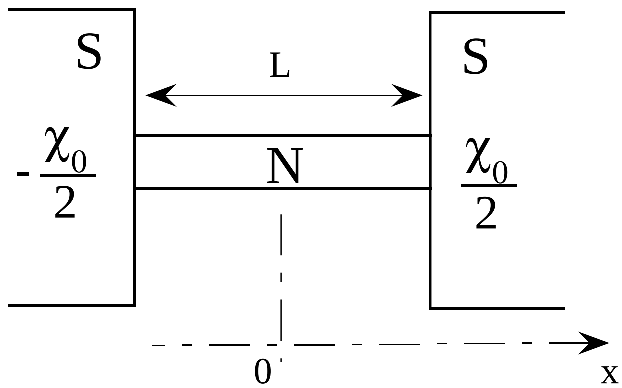

FIG. 1. The S-N-S system: the origin of coordinate x is the center of the normal metal (length L). The phases of the superconducting order parameters in the right and left superconducting electrodes are respectively $-\chi_0/2$ and $+\chi_0/2$.

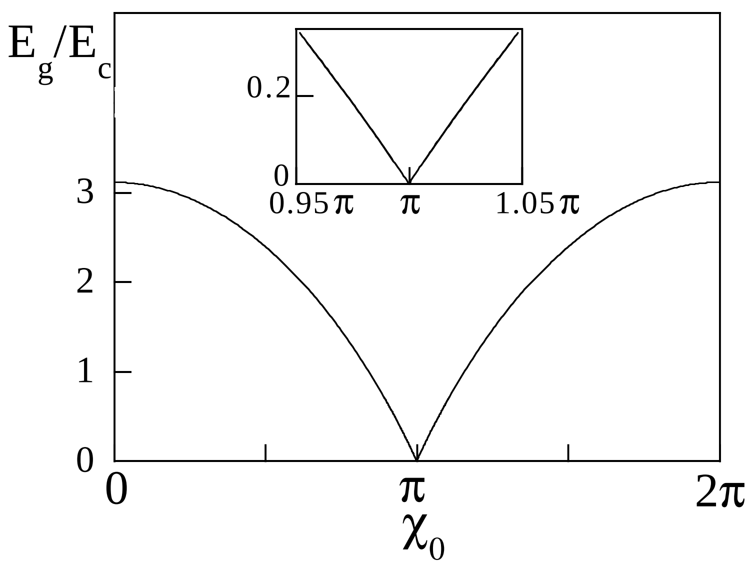

FIG. 2. Energy gap E_g in units of the Thouless energy E_c vs the phase difference χ_0 between superconducting contacts. The insert shows the linear dependence of E_g near $\chi_0 = \pi$.

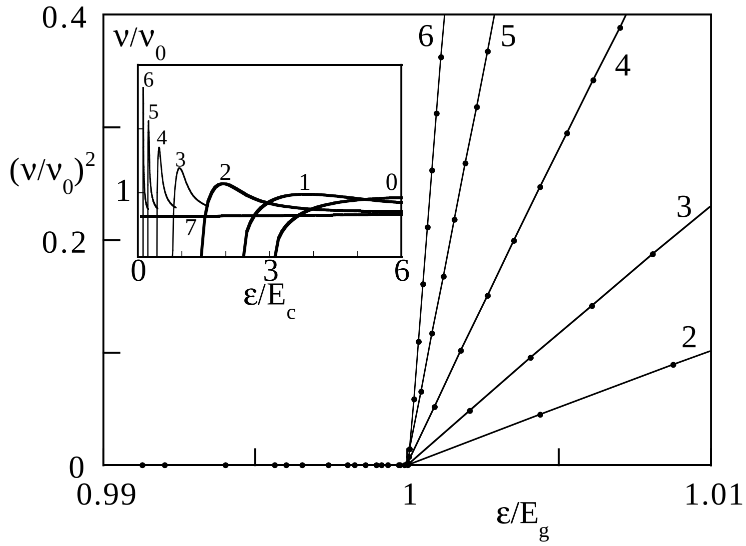

FIG. 3. The square power of the density of states vs reduced energy $\epsilon/E_g(\chi_0)$ near the gap edge. Curves labeled 0, 1, 2, 3, 4, 5, 6 and 7 are for $\chi_0 = 0$, $\pi/2$, $3\pi/4$, $7\pi/8$, $15\pi/16$, $31\pi/32$, $63\pi/64$ and π. The insert shows the full density of states curves (here $E_g \ll \Delta_0$).

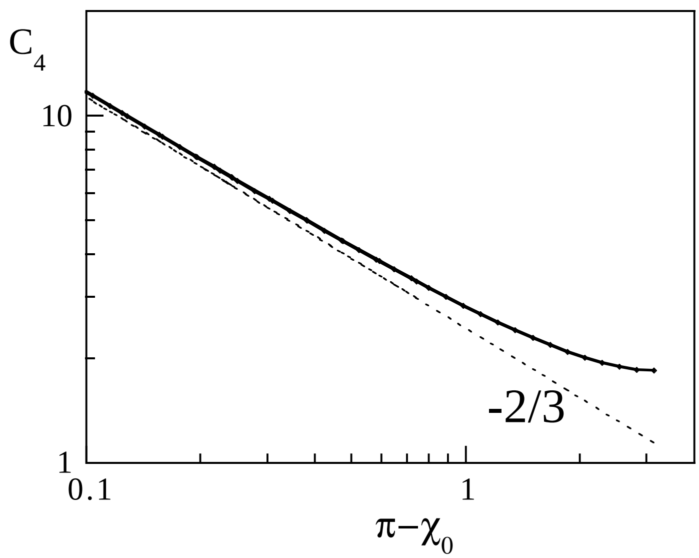

FIG. 4. Divergence of the C_4 numerical coefficient near $\chi_0 = \pi$. The dashed line is an asymptotic law $(\pi - \chi_0)^{-\beta}$ with $\beta = 2/3$.